ADAPTIVE, DYNAMICALLY CONSTRAINED PROCESS NOISE ESTIMATION FOR AUTONOMOUS ORBIT DETERMINATION

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This paper introduces two new methods to accurately estimate process noise online for robust orbit determination in the presence of dynamics model uncertainties. Common orbit determination process noise techniques include state noise compensation and dynamic model compensation, which require offline tuning, as well as covariance matching adaptive filtering, which is not dynamically constrained. To overcome these limitations, a novel approach is developed which optimally fuses state noise compensation and dynamic model compensation with covariance matching adaptive filtering. The proposed algorithms are validated through two case studies: an illustrative example and two spacecraft orbiting an asteroid.

EXTENDED ABSTRACT

In orbit determination, there are always differences between the modeled and true dynamics, which is known as process noise. The spacecraft is subject to complex forces which cannot be modeled perfectly such as gravity, solar radiation pressure, atmospheric drag, third-body perturbations, Earth tidal effects, and propulsive maneuvers. Furthermore, reduced order dynamics models are typically used for on-board applications due to computational limits. Neglecting the effects of dynamics modeling deficiencies can lead to large estimation errors as well as filter inconsistency and divergence.\(^1\)\(^2\) Modeling process noise for asteroid missions is especially challenging because the dynamical environment is poorly known a priori, and the process noise can change significantly as the spacecraft transitions between high and low altitude orbits. It is even more difficult if the mission is autonomous such as the Autonomous Nanosatellite Swarming (ANS)\(^3\)\(^4\) mission concept, which utilizes an autonomous swarm of small spacecraft to characterize an asteroid. This paper presents two adaptive and dynamically constrained process noise estimation algorithms that are applicable even when the dynamical environment is not known a priori and the process noise is time varying.

Two common techniques for modeling process noise in orbit determination are state noise compensation (SNC) and dynamic model compensation (DMC).\(^5\) SNC models uncertainty in the dynamics through accelerations that are treated as zero-mean white Gaussian processes. On the other hand, DMC augments the estimated state with fictitious or empirical accelerations which are modeled as a first order Gauss-Markov process. Both techniques assume a linear system described by

\[
\dot{x} = Ax + Bu
\]
where \( x \) is the state, \( A \) is the plant matrix, \( B \) is the process noise mapping matrix, and \( u \) is a zero-mean white Gaussian process. The covariance of \( u \) is given by \( \text{E}[u(t)u(\tau)^T] = \tilde{Q}\delta(t-\tau) \) where \( \delta \) is the dirac delta function. The matrix \( \tilde{Q} \) is propagated over the time interval \( t_0 \) to \( t \) through

\[
Q = \int_{t_0}^{t} \Phi(t,T)B\tilde{Q}B^T\Phi^T(t,T)dT
\]

(2)

to yield the process noise matrix, \( Q \). Here, \( \Phi \) is the state transition matrix and \( t - t_0 \) is the measurement update interval. In SNC and DMC, \( Q \) is determined by manually tuning \( \tilde{Q} \). To reduce the number of tuning parameters, it is usually assumed that \( \tilde{Q} \) is diagonal. A major drawback of both SNC and DMC is that the manually tuned \( \tilde{Q} \) is no longer optimal when the process noise changes. Additionally, SNC and DMC are poorly suited to scenarios where the dynamical environment is not well known a priori because the manual tuning cannot be done accurately. Genetic model compensation (GMC) was developed to adaptively tune the diagonal of \( \tilde{Q} \) online. However, GMC is not widely used due to its complicated implementation and sensitivity to multiple hyperparameters.

Another alternative for modeling process noise is adaptive filtering techniques, which can be broadly divided into four categories: Bayesian, maximum likelihood, correlation matching, and covariance matching. Covariance matching adaptive filtering (CMAF) techniques are frequently used because they are simple and computationally inexpensive. CMAF techniques typically utilize the filter innovations to tune the process noise matrix online, which allows them to adapt to time varying process noise. However, in contrast to SNC and DMC, CMAF does not dynamically constrain the process noise matrix. In other words, CMAF does not explicitly take into account that spacecraft state process noise is due to unmodeled accelerations. Furthermore, CMAF does not provide a direct estimate of the unmodeled accelerations.

This paper overcomes the aforementioned limitations of SNC, DMC, and CMAF by optimally fusing SNC and DMC with CMAF. This yields two novel, adaptive, and dynamically constrained process noise techniques called adaptive SNC (ASNC) and adaptive DMC (ADMC). The adaptability of the proposed algorithms is a significant advantage over SNC and DMC. Additionally, the proposed algorithms are more accurate than CMAF alone, especially when there are weakly observable state variables such as in angles only relative navigation. In addition to Earth-based missions, ASNC and ADMC are well suited to asteroid missions where the dynamical environment is poorly known a priori and the process noise is time varying. Another valuable use for ASNC and ADMC is to tune \( \tilde{Q} \) in advance for missions that will use SNC or DMC.

At each time step \( k \), ASNC and ADMC choose \( \tilde{Q}_k \) to best match \( Q_k \) with the CMAF estimate, \( \hat{Q}_k \). In this paper, \( \tilde{Q}_k \) is assumed to be diagonal. Since both \( Q_k \) and \( \tilde{Q}_k \) are symmetric, just the unique elements, which are contained in the lower triangular portions of the matrices, are matched. In this paper, the superscript \( lt \) indicates a vector that is formed by stacking the lower triangular elements of a matrix column-wise. It can be seen from Eqn. (2) that the process noise matrix \( Q \) is linear in \( \tilde{Q} \). Assuming \( Q \) is diagonal, \( Q^{lt}_k \) can be written as

\[
Q^{lt}_k = X_k \tilde{Q}^{diag}_k
\]

(3)

where \( X_k \) is a known matrix, and \( \tilde{Q}^{diag}_k \) is a vector composed of the diagonal of \( \tilde{Q}_k \). At each time step, the adaptively tuned value of \( \tilde{Q}_k \) is found by solving a constrained generalized least squares minimization which can be written in its most general form as

\[
\text{minimize} \quad v^T \hat{Q}^{diag}_k v \quad \text{subject to} \quad \hat{Q}^{lt}_k = X_k \tilde{Q}^{diag}_k + L_k v, \quad \tilde{Q}^{diag}_k \geq 0
\]

(4)

In addition to Earth-based missions, ASNC and ADMC are well suited to asteroid missions where the dynamical environment is poorly known a priori and the process noise is time varying.
where the second constraint ensures that $\tilde{Q}_k$ is at least positive semi-definite. Here $L_kL_k^T = W_k$ can be obtained from the Cholesky decomposition of $W_k$, and $W_k$ is the theoretical covariance of $\hat{Q}_k^{tt}$. The matrix $W_k$ is rigorously derived in this paper only using the assumptions that the system is linear and the Kalman filter is at steady state. Eqn. (4) can be efficiently solved by combining Paige’s method$^{10}$ with a nonnegative least squares algorithm such as active set and interior point methods.$^{12,13}$

To demonstrate the benefits of ASNC and ADMC, they are compared with CMAF, SNC, and DMC using two case studies. The first case study is a simple illustrative example$^{5–7}$ while the second case study looks at the more challenging scenario of two spacecraft orbiting an asteroid. For the first case study, consider a particle moving along the x-axis that is subject to an unknown perturbing acceleration in the x-direction$^{5–7}$ \( a_p = \frac{2\pi}{10} \cos \left( \frac{2\pi}{10} t \right) \) m/s$^2$. The estimated state is comprised of the particle position and velocity. For DMC and ADMC, the state additionally contains an empirical acceleration in the x-direction. Note that for this system, $\tilde{Q}$ is a scalar. Range and range rate measurements are taken from the origin every 0.1 s and are corrupted with zero mean white Gaussian noise with standard deviations of 2 m and 0.1 m/s respectively. Figure 1 shows the estimation mean absolute error (MAE) when using SNC, DMC, ASNC, and ADMC as a function of the square root of the initial guess of $\tilde{Q}$. The MAE when no process noise technique is used and when using CMAF is also shown in Figure 1 for reference.

![Figure 1](image.png)

**Figure 1**: MAE in position, velocity, and acceleration for different process noise techniques. Each MAE is computed from 100 Monte Carlo simulations.

Many interesting trends are visible in Figure 1. As expected, not using any process noise technique leads to filter inconsistency and large estimation errors. SNC and DMC perform well when an optimal value of $\tilde{Q}$ is used. However, when the value of $\tilde{Q}$ is suboptimal the estimation errors become large, and the filter can become inconsistent. Interestingly, DMC is more robust than SNC to a poor choice of $\tilde{Q}$.$^5$ CMAF maintains filter consistency, but has large position errors. Remarkably, ASNC and ADMC maintain filter consistency and achieve low estimation error regardless of the initial $\tilde{Q}$.

This paper develops two adaptive and dynamically constrained process noise techniques that have distinct advantages. ASNC is simpler to implement and less computationally expensive than ADMC. However, ADMC is slightly more accurate because the empirical accelerations yield more
accurate propagation of the mean state estimate. Additionally, ADMC provides a direct estimate of the unmodeled accelerations which may be useful depending on the application. The proposed algorithms enable accurate, autonomous navigation and are enabling technologies for asteroid missions such as the Autonomous Nanosatellite Swarming (ANS) mission concept.\cite{3,4} Note that this paper contains a detailed analysis of the additional computation time incurred by ASNC and ADMC. Future work includes estimating the off-diagonal elements of $\tilde{Q}$, modeling the empirical accelerations as a second order Gauss-Markov process, and fusing SNC and DMC with other adaptive filtering techniques. Additionally, ASNC and ADMC will be integrated with the ANS project\cite{3,4} which will include applying the algorithms to unscented Kalman filtering.

REFERENCES


