High-Fidelity Verification of Vision-Based Sensors for Inertial and Far-Range Spaceborne Navigation

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This work addresses the development, calibration and utilization of a hardware-in-the-loop testbed to verify the functionality and performance of vision-based sensors used for inertial and far-range spaceborne navigation. This testbed overcomes deficiencies of existing spaceborne vision-based navigation verification facilities by emulating the geometric and radiometric characteristics of both stellar and non-stellar objects (satellites, space debris, etc.) over a high-dynamic range while maintaining high angular accuracy. The vast majority of these facilities only simulate stellar objects used for inertial navigation. The addition of non-stellar objects into the simulation provides a hardware-in-the-loop environment to verify algorithms which abridge inertial and far-range relative vision-based navigation modes. To achieve the aforementioned, functional and performance requirements for the testbed are derived based on a known set of vision-based sensors to be verified. These requirements steer the design and development of the optical stimulator. A procedure to perform a geometric and radiometric calibration of the testbed is presented to achieve realistic emulation of the simulated space scene. Results from this calibration demonstrate the ability to simulate point sources of light to within tens of arcseconds of angular accuracy, spanning eight orders of radiometric magnitude. The calibrated testbed is then utilized to simulate static and dynamic inertial scenarios along with a far-range (angles-only) navigation scenario. These high-dynamic range, hardware-in-the-loop simulations allow for the verification of optical hardware, software and algorithms utilized by vision-based sensors for spaceborne navigation.

Key Words: optical stimulator, vision-based sensors, hardware-in-the-loop verification

Nomenclature

Superscripts and Subscripts

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<thead>
<tr>
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<th>Description</th>
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<tr>
<td>A</td>
<td>area</td>
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<td>a</td>
<td>reflectance coefficient</td>
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<tr>
<td>α</td>
<td>azimuth</td>
</tr>
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<td>c</td>
<td>principal point</td>
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<td>horizontal distortion coefficients</td>
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<td>vertical distortion coefficients</td>
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<td>DC</td>
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<td>ϵ</td>
<td>elevation</td>
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<td>focal length</td>
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<td>HD</td>
<td>Henry Draper identifier</td>
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<td>J</td>
<td>instantaneous field of view</td>
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<td>I</td>
<td>irradiance</td>
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<td>iFOV</td>
<td>loss function</td>
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<td>k</td>
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<td>μ</td>
<td>centroid location</td>
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<td>̂n</td>
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<td>r</td>
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<td>covariance matrix</td>
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<td>covariance element</td>
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<td>u</td>
<td>horizontal pixel coordinate</td>
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<td>v</td>
<td>vertical pixel coordinate</td>
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<td>X</td>
<td>horizontal pinhole projection</td>
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<tr>
<td>Y</td>
<td>vertical pinhole projection</td>
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<tr>
<td>eci</td>
<td>Earth-centered inertial</td>
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<tr>
<td>gt</td>
<td>ground truth</td>
</tr>
<tr>
<td>meas</td>
<td>measured</td>
</tr>
<tr>
<td>nso</td>
<td>non-stellar objects</td>
</tr>
<tr>
<td>os</td>
<td>optical stimulator</td>
</tr>
<tr>
<td>so</td>
<td>stellar objects</td>
</tr>
<tr>
<td>vbs</td>
<td>vision-based sensor(s)</td>
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1. Introduction

Vision-based sensors (VBS) are a ubiquitous part of the satellite navigation system. Common sensors used for inertial navigation are star trackers (ST), Sun and Earth sensors. These sensors are also extensively used for spacecraft relative navigation and in recent years have been extended to facilitate autonomous rendezvous and proximity operations. Examples of such applications are abundant in the distributed space systems community (Orion, mDOT, Exo-S, CPPOD, AVANTI, ARGON, etc.). Relative vision-based navigation techniques can be applied at inter-spacecraft separations ranging from tens of kilometers to centimeters. At large separation distances, the relative motion between spacecraft can be determined using angles-only navigation, which has been considered in several research studies. In this navigation mode, the observer spacecraft is attempting to estimate the relative orbital motion of a target space object using only bearing angles obtained by the VBS. At close range, pose estimation algorithms can be used to estimate relative position and attitude of a target space object from a single image. Close-range VBS pose estimation algorithms can utilize known visual markers and/or computer aided models of the space object to estimate the relative posi-
Testing VBS on ground has become of increasing importance for a variety of reasons. One reason is due to a paradigm shift occurring in the new space era, with miniaturization and mass production of satellites necessitating the ability to verify VBS reliably and more efficiently. A second reason stems from sophisticated applications with little to no flight heritage imposing demanding angular and radiometric sensor detection requirements. In addition, there is great interest in the engineering community in vision-based navigation systems capable of bridging the gap between far- and close-range navigation techniques. This gap comprises mixed navigation modes (i.e. performing inertial and far-range relative navigation simultaneously), transitions between navigation modes (i.e. transition from far- to close-range relative navigation) and highly-variable modes (i.e. optical navigation at highly varying separation and illumination conditions). For example, the long range VBS used on the Autonomous Rendezvous demonstration using GPS and Optical Navigation (ARGON) experiment conducted in the framework of the PRISMA (OHB) mission suffered from blooming when the NSO was in view, which impacts the quality of its inertial and relative navigation solution. Before the next generation of vision-based navigation systems can be deployed in space, a high-fidelity, high-dynamic-range testbed is necessary to properly verify algorithms, software and hardware in terms of functionality and performance.

Spaceborne VBS navigation testing facilities have historically been designed to calibrate and assess only ST. Typically, a testbed consists of a static star field and corrective optics to account for the finite distance between the OS and the camera. These systems typically have limited geometrical imaging precision, radiometric dynamic range, contrast ratio and lack the appropriate software to simulate stellar objects (SO) and non-stellar objects (NSO) in real-time and closed-loop. With these limitations in mind, it is evident that advancements to a laboratory testing environment are necessary to verify the next generation of spaceborne vision-based navigation systems.

This introduction is followed by a literature review, which seeks to identify state of the art VBS testing facilities with the intent of building off the work of others. Following this literature review, a set of functional and performance requirements are established to steer the testbed design process. Physically based geometric and radiometric models are developed for SO and NSO in the context of inertial and far-range navigation, respectively. These models then motivate a series of geometric and radiometric calibrations of the entire optical chain. It will be demonstrated that a fully calibrated system is capable of positioning a point source of light to within tens of arcseconds of angular accuracy over eight orders of radiometric magnitude. Results are analyzed and discussed prior to the conclusions and way forward. Note that although the OS is capable of simulating inertial, far- and close-range relative navigation scenarios, this paper will limit its analysis of results relating only to inertial and far-range relative navigation simulations.

2. Literature Review

Facilities which have developed the ability to stimulate VBS in a spacecraft attitude determination context have to date, primarily focused on simulating star fields used for inertial navigation sensors. Several of these facilities are briefly summarized in Table 1.

The Three-Axis Simulator 2 (TAS-2) testbed developed at the Naval Postgraduate School is a five-foot diameter simulator floating on spherical air bearings. TAS-2 was augmented with a ceiling mounted monitor to enable VBS experiments. This monitor was positioned above a camera, with no intermediate collimating optics (CO) or radiometric verification of the visual magnitude of stars being simulated.

At the John Hopkins Applied Physics Laboratory, the Celestial Object Simulator (CEL) consists of light emitting diodes (LED) attached to an acrylic hemisphere which are used to mimic the hundred brightest singlet stars in the northern hemisphere of the night sky. A 2-axis gimbal mechanism was used to slew the star camera positioned at the center of the hemisphere and simulate relative attitude motion. The use of individual LED allowed this group to emulate SO over 5 stellar magnitudes of dynamic range and was calibrated from a radiometric stand-point using a procedure relying on spectrometers and neutral density filters. Although the authors used a CO, they did not pursue quantifying the geometrical distortion it introduced.

Rufino and Moccia created a testing facility which consisted of a cathode ray tube monitor stimulating a ST through a CO. The cathode ray tube testbed (CRTT) was enclosed in a 1.8 by 0.6 by 0.6 [m] shroud with explicit calibration efforts invested to quantify the irradiance output of the monitor versus digital count. The methodology to simulate SO was to illuminate a single pixel which limited the geometrical resolution of the testbed to the instantaneous field of view (iFOV) of a monitor pixel (∼50 [arcsec]). The authors correct for aberrations introduced by the CO in software, and quantify the temporal nature of a dynamic simulation rigorously. This testbed however did not simulate NSO within the FOV.

The Jenoptik Optical Sky field Simulator (OSI) used by the German Aerospace Center (DLR) is a compact device which is attached to an arbitrary ST through a custom adapter. A simulated scene stimulates a ST with collimated light and accounts for optical distortions by warping the intended scene as well. The distortion of the CO is isolated from that of the VBS and is characterized using a pair of fourth-order calibration polynomials which results in the ability to render a single star to within ~10 [arcsec]. Pixel intensity is commanded using a Gaussian point-spread function based off star visual magnitudes. This testbed has the ability to render planetary objects (Sun, Moon, asteroids), but does not describe the ability to simulate satellites within the field of view (FOV).

The Optical Stimulator System for VBS (OSVBS) testbed created by the Technical University of Denmark (DTU) consists of a computer monitor viewed by a VBS through a CO. The testbed is large and encased in a shroud, but no other mentions of radiometric emulation are described. Geometrically, the authors account for optical distortion by rendering a scene to the monitor which is warped using openCV. The resulting attitude solutions differ in the single arc-second range from the intended attitude. The authors do not distinguish between distortion of the camera’s optical elements and the distortion introduced by the CO. This testbed has the capability to simulate
satellites within the FOV.

Limitations of previous optical testbeds consist of insufficiencies in angular accuracy, radiometric dynamic range, the ability to simulate a rapidly changing scene, accounting for geometric distortion, matching radiometric characteristics, maintaining portability and simulating multiple, mixed and transition navigation modes (i.e. inertial, far-range and close-range) simultaneously.

3. **Testbed Requirements**

The objective of this testbed is to stimulate space-capable optical hardware using synthetically created scenes which are highly representative of the space environment. The scenes of interest consist of SO and NSO, which impose their own independent set of functional and performance requirements on the testbed.

3.1. **Stellar Objects**

From a functional stand-point, the OS should produce an image realistic enough for a ST to obtain a lock and produce an inertial attitude solution based off of the observed SO. This overall goal imposes requirements on the systems ability to geometrically place a SO and simulate its radiometric characteristics. If the aforementioned requirements are met, simulated SO observed by a VBS can be identified within a star catalog. Ideally, the display for the OS should be able to simulate SO within the angular resolution and detection limit of the VBS, which for considered cameras are 10–20 [arcsec] and a visual magnitude range of 2 – 7, respectively. The relationship between visual magnitude and irradiance is given by

\[
m = -2.5 \log_{10} \left( \frac{I}{I_0} \right)
\]  

where \(m\) is the visual magnitude of the star, \(I_0\) is the reference irradiance of a visual magnitude 0 star (\(I_0 = 3.1 \cdot 10^{-9} \text{ [Wm}^{-2}]\)), and \(I_\text{so}\) is the irradiance of an observed SO.

Using Equation 1, the aforementioned visual magnitude bounds correspond to an irradiance range of \(5 \cdot 10^{-12} \text{ [Wm}^{-2}] \leq I_\text{so} \leq 5 \cdot 10^{-10} \text{ [Wm}^{-2}]\). The radiometric performance requirement associated with simulating SO imposed on the OS is to be able to radiate light over this irradiance range with a single OS monitor pixel.

3.2. **Non-Stellar Objects**

A NSO is defined to be a space object which is not a star. The introduction of a NSO into the rendered scene should take into account the objects’ attitudes, inter-object separation, material properties, and positions relative to illuminating sources (Sun, Earth, etc.) at distances ranging from the sub-meter level to tens of kilometers. With this operating envelope in mind, the rendering architecture is sub-divided into two modes: far- and close-range simulations.

At large inter-spacecraft separations, the observing vehicle does not need to distinguish fine features of the target, but instead needs to accurately determine the line-of-sight (LOS) vector to the target NSO and background SO. In this mode the observed object will resemble a point source of light. The irradiance received by the observing vehicle is modeled as the light reflected off the NSO, which is given by

\[
I_{nso} = a \left( \frac{\Omega}{2\pi} \right) I_{\text{solar}} = a \left( \frac{A}{2\pi r^2} \right) I_{\text{solar}}
\]  

where \(I_{nso}\) is the irradiance emitted by the NSO, \(\Omega\) is the solid angle subtended by the NSO, \(a\) is the reflectance coefficient of the NSO, \(I_{\text{solar}}\) is the visible solar irradiance of 620 [Wm\(^{-2}\)] at 1 astronomical unit (AU), \(A\) is the characteristic area of the NSO, and \(r\) is the inter-object separation. Note that the characteristic area of the NSO is functionally dependent on its attitude and position relative to an observer.

The NSO radiometric performance requirements imposed on the OS monitor are not only a function of characteristics of the simulated NSO (i.e. range of \(r, A, a\)), but also on the instantaneous field of view (IFOV) of the OS monitor pixels, \(\text{IFOV}_{os}\). The number of OS monitor pixels, \(N\), required to match this geometry is calculated using similar triangles, and is given by

\[
N = \frac{\sqrt{\Omega}}{\text{IFOV}_{os}}
\]

The irradiance which must be emitted by a single OS monitor pixel, \(I_{os}\), is taken to be the quotient of \(I_{nso}\) and \(N^2\).

\[
I_{os} = \frac{I_{nso}}{N^2}
\]

Equation 4 is used to compute the NSO irradiance performance requirement imposed on an OS monitor pixel. For example, consider a NSO with \(A = 2500 \text{ [cm}^2]\), \(a = 1 \cdot 10^{-3}\), over an inter-object separation ranging from 10 [m] to 100 [km], and \(\text{IFOV}_{os} = 10 [\text{arcsec}]\). The NSO irradiance, subtended solid angle, and number of monitor pixels over a range of inter-object separation are tabulated below in Table 2.

These quantities are used with Equation 4 to compute an NSO peak irradiance performance requirement imposed on a single OS monitor pixel of \(I_{os} = 2.3 \cdot 10^{-10} \text{ [W m}^{-2}]\).

<table>
<thead>
<tr>
<th>Testbed</th>
<th>Collimated Light</th>
<th>Geometric Calibration</th>
<th>Radiometric Calibration</th>
<th>Stellar Objects</th>
<th>Non-Stellar Objects</th>
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<tr>
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<td>No</td>
<td>No</td>
<td>No</td>
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<td>No</td>
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<tr>
<td>CEL(^{(19)})</td>
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<td>Yes</td>
<td>No</td>
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<tr>
<td>CRTT(^{(18)})</td>
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<td>Yes</td>
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<tr>
<td>OSI(^{(17)})</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
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<tr>
<td>OSVBS(^{(22)})</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
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</tr>
</tbody>
</table>
Table 2. NSO solid angle and irradiance for a given inter-object separation. These quantities are used with a known iFOV to compute the number of pixels required to match NSO irradiance.

<table>
<thead>
<tr>
<th>$r$ [m]</th>
<th>$\theta$ [deg]</th>
<th>$I_{nso}$ [Wm$^{-2}$]</th>
<th>$N$ [pixels]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \cdot 10^2$</td>
<td>$5 \cdot 10^{-6}$</td>
<td>$2.5 \cdot 10^{-12}$</td>
<td>$1 \cdot 10^{-1}$</td>
</tr>
<tr>
<td>$1 \cdot 10^4$</td>
<td>$5 \cdot 10^{-5}$</td>
<td>$2.5 \cdot 10^{-10}$</td>
<td>$1 \cdot 10^4$</td>
</tr>
<tr>
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<td>$5 \cdot 10^{-4}$</td>
<td>$2.5 \cdot 10^{-8}$</td>
<td>$1 \cdot 10^1$</td>
</tr>
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<td>$1 \cdot 10^2$</td>
<td>$5 \cdot 10^{-3}$</td>
<td>$2.5 \cdot 10^{-6}$</td>
<td>$1 \cdot 10^2$</td>
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<tr>
<td>$1 \cdot 10^1$</td>
<td>$5 \cdot 10^{-2}$</td>
<td>$2.5 \cdot 10^{-4}$</td>
<td>$1 \cdot 10^3$</td>
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4. Optical Stimulator Design Summary

The OS testbed consists of an organic light-emitting diode (OLED) monitor which is commanded over a video graphics array (VGA) cable by an external workstation. This OLED monitor is attached to a three-axis translational stage which is connected to a 600 [mm] by 300 [mm] optical breadboard. Light radiated by the OLED monitor stimulates a VBS mounted to the optical breadboard with post holders through a CO. The CO is housed in a threaded mount which can translate along optical rails parallel to the optical axis of the CO. COs of different focal length can readily be interchanged to stimulate VBS with different FOV. These design characteristics are depicted in the computer-aided design shown in Figure 1, and the physical realization of the OS is depicted in Figure 2.

5. Modeling Stellar and Non-Stellar Objects

This section will model the digital imaging chain which consists of radiation of light at the source, whether that be a SO or NSO, and ends with a digital count distribution at the VBS detector. The objective is to produce a high-fidelity model of this process and be able to synthetically replicate it with the OS.

5.1. Stellar Object Irradiance

The monitor of the OS should be commanded in a manner which produces a point source of light at the simulated irradiance level when observed by the VBS test article. Given knowledge of the visual magnitude of the SO to be simulated, the unknown to be computed is the irradiance of the SO to be emulated by the OS. A relationship between visual magnitude and SO irradiance was presented with Equation 1. Solving Equation 1 for the SO irradiance yields

$$I_{so} = 10^{(-m/2.512)} I_0$$

(5)

where $m$ is the visual magnitude of the SO, $I_0$ is the reference irradiance of a visual magnitude 0 star ($I_0 = 3.1 \times 10^{-9}$ [Wm$^{-2}$]), and $I_{so}$ is the irradiance of an observed SO.

5.2. Non-Stellar Object Irradiance

The irradiance emitted by a NSO is assumed to be due to light reflected off each of the vehicle’s surfaces from the Sun and Earth albedo. The solar irradiance contribution, $I_{solar}$, at 1 astronomical unit (AU) in the visible spectrum (300-700 [nm]) is approximately 623 [Wm$^{-2}$].$^{26}$ The Earth albedo irradiance contribution, $I_{albedo}$, is a complex function of position, time, and is not well standardized. This complex function is approximated by modeling the illuminated cross-sectional plane of Earth, then the illuminated area is foreshortened which reduces the albedo,

as given by

$$I_{albedo} = \max \left[ 0, a_{\text{earth}} \left( \frac{\epsilon_{nso/nso}}{2\pi r_{nso}^2} \right) I_{solar} \right]$$

(6)

where $a_{\text{earth}}$ is the albedo reflectance coefficient of Earth, $n_{nso/nso}$ is the unit vector pointing from the Earth to Sun, and $r_{nso}$ is the orbital radius of the albedo illuminated NSO. If the exterior of the vehicle is modeled as a collection of planar panels, then the total irradiance from the NSO is the sum of each visible panels’ reflecting contribution, given by

$$I_{nso} = \sum_{i=1}^{n} \epsilon_i a_i \left( \frac{n_i^T n_{nso/nso}}{2\pi r_i^2} \right) I_{solar} + \left( \frac{A_i (n_i^T n_{nso/nso})}{2\pi r_i^2} \right) I_{albedo}$$

(7)

where $n$ is the number of panels, $\epsilon_i$ is a binary flag (0 or 1) depending on if the panel is visible to the observing VBS, $a_i$ is the panel’s reflectance coefficient, $A_i$ is the surface area of the $i^{th}$ panel, $r_i$ is the distance separating the $i^{th}$ panel from the observing camera, $n_i, n_{nso/nso}$ are the panel’s outward surface unit normal and the unit vector pointing from the panel to the Sun. Some of the limiting assumptions used to simplify the complexity of the NSO irradiance model are the neglect of shadows on the NSO and a constant Earth albedo of 0.3.$^{25}$ Equation 7 could be augmented to include the aforementioned terms.

5.3. Gaussian Point Spread Function

The calculated irradiance of SO and NSO presented in the previous two sections may be greater than the irradiance output of a single monitor pixel. When this occurs, the irradiance of the object being simulated, $I$, can be matched by super-imposing the irradiance contribution of multiple adjacent monitor pixels. A multi-variate Gaussian point spread function (PSF) is used to emulate the desired irradiance level while simultaneously ensuring the centroid of the observed SO/NSO is at the proper angular location. The digital count distribution commanded to the monitor/detector to achieve this PSF is given by

$$DC(x) = g \left( \frac{1}{2\pi \sqrt{\det(\Sigma)}} \exp \left(-\frac{1}{2} (x - \mu)^T \Sigma^{-1} (x - \mu) \right) \right)$$

(8)

where $x = \left[ \begin{array}{c} u \\ v \end{array} \right]$ is an arbitrary pixel location, $\mu = \left[ \begin{array}{c} \mu_u \\ \mu_v \end{array} \right]$ is the centroid of the Gaussian, $\Sigma$ is the covariance matrix, and the empirical function $g$ maps the desired irradiance to its corresponding digital count, DC. The function $g$ and the elements of the covariance matrix are unknowns which are obtained through a calibration process presented in the next section. Additionally, the centroid of the Gaussian, $\mu$, is also unknown.

5.4. Geometric Placement

The selection of the Gaussian centroid, $\mu$, must produce a collimated beam of light reaching the aperture of the VBS test article. For SO, this centroid location is a function of the attitude of the camera relative to an inertial reference frame, unit vectors derived from the Hipparcos star catalog, and stellar aberration calculations. For NSO, orbit propagation is used to calculate the relative separation between NSO and the observing VBS (see Appendix A).
The simulated angular location of the point source of light can be connected to $\mu$ by performing a ray tracing calculation through the CO. This approach is computationally expensive and is not preferred. Alternatively, the geometric placement of a Gaussian PSF on the OS monitor can be calculated using the thin lens equation with the OS monitor placed at the focal point of the CO. Under the thin lens assumptions, a ray of light passing through the center of the CO will undergo no refraction. Since the light exiting the CO is assumed to be collimated, all other rays of light must be parallel to the aforementioned ray, as seen in Figure 3. Therefore, to simulate a collimated point source of light at horizontal/vertical angular location $\theta_x, \theta_y$, the OS monitor centroid should be selected with

$$\mu = f_{co} \left[ \tan \theta_x \ \tan \theta_y \right]^T + c_0 \quad (9)$$

where $c_0$ is the center location of the OS monitor in pixels.

Although Equation 9 is trivial to implement, its neglect of optical aberration will introduce deviations between the intended and measured angular location of point sources of light which are unacceptable. The next section introduces a geometric calibration procedure to identify what monitor pixels in the presence of CO distortion should be illuminated to simulate a point source of light without having to perform computationally expensive ray tracing.

6. Calibration of Optical Stimulator

The purpose of calibrating the OS is to place precise radiometric point sources of collimated light at the intended angular location in the presence of optical distortions and brightness attenuating/dispersive mechanisms. The geometric portion of the calibration characterizes the aberration effects present in the camera and CO, while the radiometric portion quantifies the function which maps a monitor digital count to irradiance and the covariance scaling transformation between PSF rendered to the monitor and observed on the VBS detector. The overall calibration strategy is summarized in the following steps

1. Independently calibrate a VBS known as the calibration article to measure un-distorted unit vectors to point sources of light.
2. Estimate the intrinsic parameters of the CO and extrinsic parameters of the calibration article mounted to the OS.
3. Build empirical function mapping digital count on OS monitor to irradiance exiting CO.
4. Build empirical function mapping monitor covariance to calibration article covariance.
5. Estimate mounting of a vision-based sensor known as the test article using $q$-method.26)

6.1. Geometric

The objective of the geometric calibration is to identify the OS monitor pixel centroid, $\mu$, which in the presence of distortion from the CO, stimulates the VBS test article from the intended angular location. This is a difficult problem to solve because quantifying the distortion of the CO requires the use of a VBS with its own set of distorting optics. To isolate the
distortion of the VBS from that of the CO, the geometric calibration procedure is divided into two parts: 1) independently calibrating a VBS known as the calibration article and then estimating the distortion parameters of the CO. It will be shown that an independently calibrated VBS can isolate the distortion introduced by the CO. After the distortion of the CO is quantified, the scene rendered to the OS monitor is warped to achieve the aforementioned objective.

### 6.1.1. Calibration Article

The purpose of the independent calibration is to identify the intrinsic parameters which characterize the mapping of features in 3D space to the 2D detector of the calibration article. At the completion of the calibration process, un-distorted detector measurements on the calibration article can be used to formulate the 3D camera coordinates associated with an observed point source of light. This capability is a necessary tool to isolate and quantify the distortion introduced by the CO. A geometrical model used to characterize the calibration article is the second-order pinhole-distortion model\(^{(27)}\) given by

\[
\begin{align*}
\mathbf{u} &= f_{\text{vbs}}[(1 + k_1 r^2 + k_2 r^4) X + 2k_3 X Y + k_4 (r^2 + 2 X^2)] + c_x \\
\mathbf{v} &= f_{\text{vbs}}[(1 + k_1 r^2 + k_2 r^4) Y + k_3 (r^2 + 2 Y^2) + 2k_4 X Y] + c_y
\end{align*}
\]

(10)

(11)

where \(u, v\) are the 2D pixel coordinates of a feature on the camera detector in the horizontal/vertical directions, \(c_x, c_y\) are the coordinates of the principal point, \(X, Y\) are the horizontal/vertical pinhole projections, \(r\) is the radial distance from the principal point, \(k_1, k_2\) are the radial distortion coefficients, and \(k_3, k_4\) are the tangential distortion coefficients. If both the 3D spatial and 2D coordinates associated with a set of features are accurately known, the intrinsic optical parameters can be estimated using a least-squares fit. Equation 11 can be rearranged into a set of linear equations of the form \(\mathbf{b} = \mathbf{A}\mathbf{x}_{\text{est}}\) where

\[
\mathbf{A} = \begin{bmatrix}
X_1 & r_1^2 X_1 & r_1^2 Y_1 & 2X_1 Y_1 & r_1^2 + 2X_1^2 & 1 & 0 \\
Y_1 & r_1^2 Y_1 & r_1^2 Y_1 & Y_1 & r_1^2 + 2Y_1^2 & 2X_1 Y_1 & 0 & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
X_n & r_n^2 X_n & r_n^2 Y_n & 2X_n Y_n & r_n^2 + 2X_n^2 & 1 & 0 \\
Y_n & r_n^2 Y_n & r_n^2 Y_n & Y_n & r_n^2 + 2Y_n^2 & 2X_n Y_n & 0 & 1
\end{bmatrix}
\]

\[
\mathbf{b} = \begin{bmatrix}
u_1 \\
u_2 \\
\vdots \\
\vdots \\
u_n
\end{bmatrix}
\]

\[
\mathbf{x}_{\text{est}} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{b}
\]

(12)

(13)

With a sufficient number of measurements, \(n\), the left-inverse of \(\mathbf{A}\) can be taken to obtain the estimates of the intrinsic parameters, \(\mathbf{x}_{\text{est}}\), as seen below

\[
\mathbf{x}_{\text{est}} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b}
\]

Note that the estimated products (second through fifth elements of \(\mathbf{x}_{\text{est}}\)) must be divided by the focal length of the VBS to obtain the true distortion coefficients. For this estimation process, the matrix \(\mathbf{A}\) is populated with quantities from a set of reference calibration features with known 3D coordinates, and the vector \(\mathbf{b}\) is populated with the measured 2D calibration article detector feature locations.

To illustrate the population of \(\mathbf{A}\) and \(\mathbf{b}\), a calibration procedure utilizing stars will be described next. This procedure is applied to two cameras referred to as the calibration and test articles, whose specifications are outlined in Table 3. The calibration article is independently calibrated and used to estimate the intrinsic parameters of the CO, while the test article is the VBS being simulated by the OS for verification purposes.

**Table 3. VBS specifications of calibration and test article.**

<table>
<thead>
<tr>
<th>Property</th>
<th>Calibration Article</th>
<th>Test Article</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor</td>
<td>Aptina AR0331</td>
<td>Sony IMX174</td>
</tr>
<tr>
<td>Resolution</td>
<td>2048 x 1536</td>
<td>1920 x 1200</td>
</tr>
<tr>
<td>Pixel Size</td>
<td>2.2 [μm]</td>
<td>5.86 [μm]</td>
</tr>
<tr>
<td>Focal Length</td>
<td>17 [mm]</td>
<td>34.9 [mm]</td>
</tr>
<tr>
<td>F-number</td>
<td>1.4</td>
<td>1.9</td>
</tr>
<tr>
<td>Transmission</td>
<td>400-1000 [nm]</td>
<td>400-1000 [nm]</td>
</tr>
<tr>
<td>iFOV</td>
<td>26.5 x 26.6 [arcsec]</td>
<td>34.2 x 34.5 [arcsec]</td>
</tr>
</tbody>
</table>

**Star Calibration** To motivate the use of the aforementioned least squares problem, an experimental setup to calibrate a VBS is analyzed using images of the starry night sky. Stars make for ideal calibration features because their angular location with respect to Earth is accurately known. By processing an image of the starry night sky with a star identification algorithm one can obtain several important pieces of information for each star in the FOV of the VBS such as the location of the \(i^{th}\) star centroid on the detector, \(u_i, v_i\), its corresponding reference star catalog unit vector in Earth centered inertial (ECI) coordinates, \(\mathbf{x}_{\text{ref}}\), and a statistical estimate of the direction cosine matrix (DCM) between the VBS and ECI frames, \(\mathbf{R}_{\text{eci}}\). This last piece of information is used to re-express the reference star unit vectors in VBS coordinates, \(\mathbf{v}_{\text{bs}}\mathbf{x}_{\text{ref}}\), as given by

\[
\mathbf{v}_{\text{bs}}\mathbf{x}_{\text{ref}} = \mathbf{R}_{\text{eci}}\mathbf{x}_{\text{ref}}
\]

(14)

After re-expressing the inertial unit star vectors in the VBS frame, there will be some residual deviation between the reference and measured unit vectors. This deviation is characterized with the intrinsic camera parameters in the vector \(\mathbf{x}_{\text{est}}\). By taking hundreds of pictures of the night-sky, many measurements were used to populate the the matrix \(\mathbf{A}\) and vector \(\mathbf{b}\) to obtain the least-squares estimate of the intrinsic camera parameters.

The calibration article can now report un-distorted unit vectors of observed point sources at pixel location \(u, v\) by solving Equation 11 for \(X\) and \(Y\). These un-distorted unit vectors will play a critical role in geometrically calibrating the CO. If the tangential distortion coefficients are sufficiently small, then the solution for the horizontal/vertical pinhole projections in Equation 11 simplify to

\[
X = \frac{1}{1 + k_1 r^2 + k_2 r^4} \left(\frac{u - c_x}{f_{\text{vbs}}}\right)
\]

(15)

\[
Y = \frac{1}{1 + k_1 r^2 + k_2 r^4} \left(\frac{v - c_y}{f_{\text{vbs}}}\right)
\]

(16)

and the corresponding unit vector, \(\hat{\mathbf{n}}\), is
\[ \hat{n} = \begin{bmatrix} X & Y & 1 \end{bmatrix}^T \quad (17) \]

### 6.1.2. Collimating Optic

As mentioned earlier, the optical aberration introduced by the CO will limit the useful applicability of Equation 9. The purpose of the geometric calibration of the CO is to identify what OS monitor pixel should be illuminated to stimulate the VBS test article from the simulated angular origin without performing any ray tracing. The model used to characterize the mapping of a point source of light with unit vector \( \hat{n} \) (from Equation 17) to an OS monitor pixel through the distortion of the CO is given with a pair of fourth order polynomials. This method was adopted from the calibration technique performed for the OSI testbed\(^{19} \) and is given by

\[ u_{\text{os}} = \sum_{i=0}^{4} \sum_{j=0}^{4} c_{i,j} X^i Y^j \quad (18) \]

\[ v_{\text{os}} = \sum_{i=0}^{4} \sum_{j=0}^{4} d_{i,j} X^i Y^j \quad (19) \]

where \( u_{\text{os}}, v_{\text{os}} \) are OS monitor coordinates, and \( j, k \) are indices which sweep over one of the 25 elements in the matrices of distortion coefficients \( c, d \). In a manner analogous to the VBS calibration procedure, if both the angular location \((X, Y)\), as well as the OS monitor feature locations \((u_{\text{os}}, v_{\text{os}})\) are known, a set of linear equations can be formulated to relate the aforementioned quantities. The distortion coefficients are estimated with linear equations of the form \( U = A c \) and \( V = A d \), where

\[
A = \begin{bmatrix} X_0^0 Y_0^0 & X_0^1 Y_1^0 & \ldots & X_0^n Y_n^0 \\ X_1^0 Y_0^1 & X_1^1 Y_1^1 & \ldots & X_1^n Y_n^1 \\ \vdots & \vdots & \ddots & \vdots \\ X_n^0 Y_0^n & X_n^1 Y_1^n & \ldots & X_n^n Y_n^n \\ \end{bmatrix} \quad (20)
\]

\[
U = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} \quad (21)
\]

\[
V = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \quad (22)
\]

With a sufficient number of measurements, \( n \), the left-inverse of \( A \) can be taken to obtain the estimates of the CO distortion coefficients as seen below.

\[
c = (A^T A)^{-1} A^T U \quad (23)
\]

\[
d = (A^T A)^{-1} A^T V \quad (24)
\]

For this estimation process, the matrix \( A \) is populated with measured quantities derived from the calibration article reporting unit vectors using Equation 17, the vectors \( U \) and \( V \) are populated with the commanded 2D OS monitor feature locations of a known calibration pattern (i.e. grid of dots). After these coefficients have been successfully estimated, one can use Equations 18-19 to calculate what OS monitor pixels to illuminate to simulate a point source at an angular location corresponding with \( X, Y \).

The geometrical impact of the CO calibration procedure is shown in Figures 4 and 5. The contours of these plots represent the angular differences in arcseconds between the reference set of monitor calibration features and the observed calibration article features before and after calibrating for the CO. The quality of the CO calibration is quantified through the magnitude of the residuals depicted in Figure 5, which are fundamentally related to the iFOV of the VBS calibration article. The angular range of residuals in Figure 5 correspond to 4% – 30% of the iFOV associated with a calibration article pixel.

### 6.1.3. Mounting

After obtaining the coefficients which characterize the distortion of the CO, point sources of light can be rendered to the monitor of the OS to produce collimated light at the aperture of a VBS test article. When a new test article is mounted in the OS, its mounting relative to the collimated field needs to be identified. Note that the position of the VBS within the field does not matter since the light is collimated. In order to calculate the misalignment, a grid of dots is rendered to the OS monitor and observed with the calibration article whose detector has reached thermal steady state. The reference unit vectors and the measured unit vectors of the VBS centroided features would ideally be identical if not for the misalignment between the VBS and monitor. This misalignment is modeled as a rigid body rotation, and solved for in a statistically optimal way with \( q \)-method.\(^{26} \) To be explicit, the orientation of the VBS bases with respect to the monitor minimizes the loss function given by
where \( J \) is the loss function to be minimized, \( R_{\text{mon}}^{\text{vbs}} \) is the direction cosine matrix (DCM) between monitor and VBS frames, \( {v}_{\text{mon}}^{\text{vbs}} \) and \( {v}_{\text{vbs}}^{\text{mon}} \) are unit vectors corresponding to the VBS and OS monitor features in their respective coordinates. After \( R_{\text{mon}}^{\text{vbs}} \) is estimated, the scene rendered to the monitor can be rotated to stimulate the VBS from the intended angular location.

6.2. Radiometric

The objective of the radiometric calibration is to characterize the functional relationship between the scene rendered to the OS monitor and the VBS detector response. This relationship is characterized through an assessment of the irradiance and covariance transformation characteristics of the OS.

6.2.1. Irradiance

A high-resolution hand-held optical power/energy meter (model 1918-R, Newport) was placed at the exit of the CO and used to quantify the radiometric flux that would arrive at the camera aperture. Measurements were taken with blocks of \( N^2 \) pixels illuminated (where \( N \) is the side length of the block in pixels) with varying digital count. This data is plotted in Figure 6. The noise floor was removed from each of these curves and then normalized by the number of pixels, \( N^2 \), to identify the functional relationship between the radiometric flux of a single pixel and commanded digital count. This procedure is given by

\[
J(\mathbf{R}_{\text{mon}}^{\text{vbs}}) = \sum_{i=1}^{n} \| {v}_{\text{mon}}^{\text{vbs}} - R_{\text{mon}}^{\text{vbs}} {v}_{\text{mon}}^{\text{vbs}} \|^2 
\]

where \( J \) is the loss function to be minimized, \( R_{\text{mon}}^{\text{vbs}} \) is the direction cosine matrix (DCM) between monitor and VBS frames, \( {v}_{\text{mon}}^{\text{vbs}} \) and \( {v}_{\text{mon}}^{\text{vbs}} \) are unit vectors corresponding to the VBS and OS monitor features in their respective coordinates. After \( R_{\text{mon}}^{\text{vbs}} \) is estimated, the scene rendered to the monitor can be rotated to stimulate the VBS from the intended angular location.

6.2.2. Covariance

The objective of this section is to obtain a mapping from PSF rendered to the OS monitor and PSF detected by the calibration article. In the most rigorous sense, this mapping should capture testbed characteristics which influence the PSF observed by the calibration article such as the angular span of a OS monitor pixel, aberrations of the CO and exact inter-component separations/orientations. The combination of these individual effects is approximated with a linear mapping between the covariance of a PSF rendered to the OS monitor and observed by the calibration test article. The construction of this linear mapping relies on the ability to quantify the parameters describing each PSF. To handle this, a mathematical model of a PSF is presented.

Expanding equation 8 yields

\[
\text{DC}(x) = \frac{1}{2\pi \sqrt{\sigma_{11}^2 + \sigma_{22}^2}} \exp \left[ -\frac{a(x - \mu_x)^2 + b(x - \mu_x) + c}{2(x - \mu_x)^2} \right]
\]

where \( \sigma_{11}, \sigma_{12}, \sigma_{22} \) are the elements of the symmetric covariance matrix, \( \Sigma \), and the coefficients \( a, b, c \) are defined as

\[
a = (v - \mu_v)^2 \\
b = (u - \mu_u)^2 \\
c = (u - \mu_u)(v - \mu_v)
\]

The centroid of the region of interest (ROI) is calculated using the first area moment given by

\[
\mu_u = \frac{\sum u_i \text{DC}(x)}{\sum \text{DC}(x)} \\
\mu_v = \frac{\sum v_i \text{DC}(x)}{\sum \text{DC}(x)}
\]

Fig. 6. Total irradiance measured at the exit of CO plotted against OS monitor digital count.

Fig. 7. Normalized irradiance measured at the exit of CO plotted against OS monitor digital count.
The covariance elements are unknowns of the VBS detector response which need to be estimated. These parameters are identified by using a Gauss-Newton method to iteratively update an estimate of the covariance elements, $\sigma$. The iterative update attempts to minimize the magnitude of the residual function, $r$, given by

$$ r(\mu, \Sigma) = DC - DC(\mu, \Sigma) $$

(33)

where DC is the actual detector response and $DC(\mu, \Sigma)$ is the modeled response of the detector based on the calculated centroid and estimated covariance elements. Using a linearized Taylor series expansion, the residual function at some perturbation from the covariance estimate can be expressed as

$$ r(\Sigma + \delta) = r(\Sigma) + r(\Sigma)\delta $$

(34)

where $\delta$ is the perturbation to the state estimate and $r$ is a matrix of partial derivatives given by

$$ r = \begin{bmatrix} \frac{\partial r}{\partial \sigma_{11}} & \frac{\partial r}{\partial \sigma_{12}} & \frac{\partial r}{\partial \sigma_{22}} \end{bmatrix} $$

(35)

The update which minimizes the magnitude of the residual function is the least-squares solution of Equation 34 for $\delta$. The update to the covariance estimate is thus given by

$$ \begin{bmatrix} \sigma_{11}^{k+1} \\ \sigma_{12}^{k+1} \\ \sigma_{22}^{k+1} \end{bmatrix}_{k+1} = \begin{bmatrix} \sigma_{11}^{k} \\ \sigma_{12}^{k} \\ \sigma_{22}^{k} \end{bmatrix}_k - \left( r^T r \right)^{-1} r^T r \begin{bmatrix} \sigma_{11}^{k} \\ \sigma_{12}^{k} \\ \sigma_{22}^{k} \end{bmatrix}_k $$

(36)

where $k$ and $k+1$ are indices corresponding to the original and updated estimates of the covariance elements.

If the VBS detector response is assumed to be a symmetric multi-variate Gaussian distribution (i.e. $\sigma = \sigma_{11} = \sigma_{22}$ and $\sigma_{12} = 0$) then the Gauss-Newton update simplifies to only estimating $\sigma$, as given by

$$ \sigma_{k+1} = \sigma_k - \left( r^T r \right)^{-1} r^T r(\mu, \sigma_k) $$

(37)

where $\dot{r}$ is given by

$$ \dot{r} = -\frac{\sigma + a + b}{4\sigma^2} \exp\left( -\frac{a + b}{\sigma^2} \right) $$

(38)

With this estimation framework in place, the empirical PSF mapping between OS monitor and calibration article is ready to be calculated. A series of symmetric Gaussian PSF are rendered to the OS monitor, holding the monitor irradiance output constant. Observation of these PSF by the calibration article stimulates a detector response with a different PSF, whose covariance parameter is estimated using Equations 37 and 38. By repeating this procedure for many different monitor irradiance levels, a mapping is constructed over the entire radiometric operating envelope of the OS.

7. Verification of Optical Stimulator

Once the OS has been calibrated the testbed has the ability to simulate precise radiometric point sources of collimated light at an intended angular location. Prior to utilizing the OS for the characterization of VBS, a set of verification analyses are performed to affirm functionality and the quality of the previous calibration steps.

7.1. Geometric Verification

Functional and performance verification of the geometric calibration of the OS was performed with two separate tests. The functional test utilized flight data from the PRISMA mission to verify that the OS qualitatively is placing SO and NSO in the correct geometric locations. The flight data utilized consisted of ARGON images taken by the Mango far-range VBS of SO and Tango (a NSO), precise orbit determination (POD) products accurate to the centimeter level in relative position, and an inertial navigation solution coming from star trackers.\(^1\)\(^{,}\)\(^2\)\(^8\) The software used to synthesize images for the OS computed the geometric location of SO and NSO with the aforementioned inputs. The OS calculated SO and NSO in view was then superimposed onto the corresponding ARGON flight image depicted in Figure 8.

Fig. 8. ARGON flight image from PRISMA mission of Tango acquired by Mango far-range VBS at 14 [km] separation on 2012-04-25.\(^1\) The geometric location of SO calculated by the OS are super-imposed over the flight image and annotated with Henry Draper (HD) identifiers. POD flight products are used by the OS to calculate the geometric centroid of the NSO. Functionally, both SO and NSO predicted by the OS match up with the PRISMA flight data and imagery.

The alignment of synthetic OS features with NSO and SO in the PRISMA flight image indicate that the OS is functionally able to render SO and NSO with geometric consistency, but it does not indicate the accuracy to which these features can geometrically be placed. For this purpose, a separate performance verification test is developed to quantify the geometric accuracy of the OS. The performance verification test renders a warped grid of dots to the OS monitor which stimulates the calibration article. The warped monitor location of these dots are computed with Equation 18-19 based off a desired set of unit vectors, $\hat{n}_{des}$, to stimulate the calibration article from. The measured unit direction, $\hat{n}_{meas}$, of stimulus is computed using Equation 17. For
each of the features observed by the calibration article, an angular residual, $d\theta$, between the desired and measured unit vectors was computed using Equation 39.

$$d\theta = \cos^{-1} \left( \hat{n}_{des} \cdot \hat{n}_{meas} \right)$$ (39)

These angular residuals were computed for each verification dot. Statistics associated with the angular residuals are summarized in Table 4, and a plot of the angular residuals cumulative distribution function (CDF) is depicted in Figure 9. These results indicate that point sources of light are stimulating the VBS from the intended angular location (in the presences of CO aberrations) at levels of accuracy which are less than a fraction of a calibration article pixel. These geometric verification results are also quantitatively consistent with the CO calibration residuals depicted in Figure 5.

Table 4. Statistics on angular residuals, $d\theta$, from the OS geometric verification. The mean and standard deviation of the experimentally computed angular residuals are given by $\bar{d}\theta$ and $\sigma$, respectively. All units are in [arc-sec].

<table>
<thead>
<tr>
<th>$d\theta$</th>
<th>1$\sigma$</th>
<th>3$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0878</td>
<td>2.9422</td>
<td>8.8265</td>
</tr>
</tbody>
</table>

![Cumulative distribution function of the angular residual, $d\theta$.](image)

---

8. Results

With a calibrated and verified OS, the testbed is ready to simulate both static and dynamic scenarios consisting of SO and NSO. The architecture used to render a scene to the OS monitor is depicted in Appendix A (see Figure 18). The outputs of the calibration section are the intrinsic parameters of the CO, extrinsic parameters of the VBS test article mounting with respect to the OS testbed, a digital count to irradiance mapping, and covariance characterizations of the OS monitor. These parameters are used to render a scene to the OS monitor which best places point sources of light at the intended angular location while matching irradiance and PSF characteristics. The angular location of SO and NSO are derived from a high-fidelity orbital and attitude simulation. Irradiance characteristics of the SO are derived from the Hipparcos star catalog, standardized tables, and astronomical references.

8.1. Static Inertial Navigation Simulation

A static star field was rendered to the OS monitor and used to stimulate a ST test article with identified mounting. The unit immediately returned a lock corresponding to the simulated star field with an angular offset. This angular offset was encoded with a DCM given by

$$R_{gt}^{\text{meas}} = R_{eci}^{\text{meas}} \left( R_{gt}^{\text{eci}} \right)^T$$ (40)

where $R_{gt}^{\text{eci}}$ is the ground truth OS simulated vehicle attitude with respect to ECI, $R_{gt}^{\text{meas}}$ is the vehicle attitude with respect to ECI measured by the ST, and $R_{gt}^{\text{meas}}$ is the ST measured attitude with respect to the OS simulated attitude. If the two former attitudes are identical, then $R_{gt}^{\text{meas}}$ will be identity. Note that the star field rendered to the monitor is a function of both the simulated attitude and the mounting DCM calculated by minimizing Equation 25.

The ST observed a static star field for 60 minutes, with $R_{gt}^{\text{meas}}$ being calculated at each ST measurement. The DCM $R_{gt}^{\text{meas}}$ was then re-expressed using its equivalent 321 Euler angle sequence representation. These Euler angles represent angular residuals between the ground truth simulation and measured vehicle attitude about axes aligned with the VBS test article and are plotted in Figure 11. These residuals undergo a noticeable transient evolution about the horizontal axis of the VBS, $\theta_z$, during the first thirty minutes which was observed to have an exponential time constant correlated with the detector’s temperature, as shown in Figures 11 and 12. As is typical with ST, the angle encoding the rotation about the boresight of the camera, $\theta_y$, has the largest variance. The solution additionally expresses a noticeable banding characteristic, which is a consequence of the attitude determination process alternating between observed stars used in the star identification process producing slightly different attitudes.

8.2. Dynamic Inertial Navigation Simulation

The static inertial navigation simulation was extended to a dynamic simulation by rendering sequential images to the OS monitor. The synthetic images emulate a scenario of a VBS at-
Optical Stimulator

Night Sky

Fig. 10. Images acquired by calibration article being stimulated by OS monitor and night sky.

Fig. 11. Angular residuals between VBS measured and simulated attitude.

Fig. 12. Temperature of the VBS detector during the static inertial navigation simulation.

Attached to a spacecraft in a circular orbit. The spacecraft controls its attitude so that it remains fixed in a rotating reference frame centered about the servicer spacecraft. In this reference frame, the $x$-axis is aligned with the zenith (R) direction, the $z$-axis is aligned with the angular momentum of the vehicle’s orbit (N) direction, and the $y$-axis is aligned with the vehicle flight (T) direction, completing the right-handed RTN triad. The VBS with fixed boresight alignment on the chief spacecraft is considered to be mounted in the anti-flight direction. With this definition, the VBS triad will undergo a constant angular velocity rotation about the N-axis of the RTN triad.

Images corresponding to this scenario definition were created a-priori and then rendered to the OS monitor in an open-loop fashion through Simulink with custom written s-functions. The refresh rate of the OS monitor is listed to be within 30-85 [Hz] which corresponds to a duration between sequential updates in the range of 12.5-33.3 [ms]. With this duration in mind, the ground-truth time-step between sequential synthetic images was selected to be 50 [ms]. To ensure the a-priori produced images were rendered at the intended time, an s-function was written to perform time synchronization with the Simulink simulation and the clock of the host workstation. This time synchronization block consists of a conditional while loop preventing execution of downstream code. The condition to exit the while loop is for a configurable amount of time to have passed, which is determined by a query to the internal oscillator of the host workstation. Additional s-function blocks within the simulation interface with the VBS test article and trigger image acquisitions. The interface to OS monitor and VBS along with the software to synchronize Simulink with the workstation clock all exist within a single simulation. This allows the timestamp of the acquired images to be compared directly to the ground-truth for assessing VBS solution quality and functional capabilities as a function of the inertial angular velocity of the sensor.
A series of three inertial dynamic experiments were conducted with different ground truth angular velocities. Unit vector measurements were derived from the HIL acquired images. An example of these images is depicted in Figure 13. Equation 39 was used to compute angular residuals between the measured and ground truth unit vectors over the duration of the experiment. The interior angle associated with these inner-products is on the order of tenths of arcseconds for the three conducted experiments, and is plotted in the cumulative distribution function (CDF) plot in Figure 14.

![Fig. 13. HIL acquired images from the inertial dynamic simulation with 0.5 \[\text{deg/s}\] angular velocity slew.](image)

The CDF plotted in Figure 14 reveals many interesting facets about the VBS test article. Observe that simulations corresponding to lower ground truth angular velocities have smaller angular residuals for all values of the CDF function. The CDF corresponding to the low angular velocity simulations has a smoothly varying nature, which is hypothesized to be a result of the image processing algorithm behaving more reliably with a slowly varying angular location of the integrated point source of light. As the angular velocity of the inertial dynamic simulation increases, the VBS test article observes stars with larger streaks, as seen in Figure 13. As the signal to noise ratio of these streaking star decreases, the quality of the image processing algorithm correspondingly degrades. The manifestation and quantification of this phenomenon is all encoded in Figure 14. The ability to simulate these scenarios on the ground has enormous potential to facilitate procedural decisions for commissioning of spacecraft, safe-modes, and nominal dynamic operations.

![Fig. 14. Cumulative distribution function of angular residuals computed for dynamic inertial navigation simulation.](image)

8.3. Dynamic Relative Navigation Simulation

With both static and dynamic inertial navigation capabilities verified using the OS, the final test scenario considered in this work looks at the problem of dynamic relative navigation of the observing spacecraft with respect to a NSO in near-circular, low Earth orbit (LEO). In this configuration, the observer spacecraft is attempting to estimate the relative orbital motion of the target space object using only bearing angles obtained by the VBS. This so-called angles-only navigation has been considered in several research studies,\(^{1-13}\) and utilized for relative navigation and rendezvous in both the ARGON experiment,\(^1\) as well as the Autonomous Vision Approach Navigation and Target Identification (AVANTI) experiment,\(^{22}\) taking place during the Firebird mission (DLR). Angles-only relative navigation represents an especially difficult estimation scenario due to the inherent dynamical observability constraints imposed by using bearing angle measurements (2D) to reconstruct the full relative orbital motion state (6D). The stability and performance of algorithms developed for angles-only navigation can be verified with greater confidence using the OS since it introduces a higher degree of realism in the verification process (i.e., the use of a real sensor in the loop) than pure software simulation methods.

For this paper, the NSO relative orbital motion is chosen to recreate one of the scenarios considered by Sullivan et al.\(^{11}\) in the context of angles-only rendezvous in LEO. In that work, the authors use a set of relative orbital elements (ROE) consisting of the relative semi-major axis ($\delta a$), the relative mean longitude ($\delta \Omega$), and the relative eccentricity and inclination vectors ($\delta e$ and $\delta i$) to parameterize the relative motion of the NSO with respect to the observing spacecraft.\(^{15}\) The initial conditions for the observing spacecraft and the relative motion of the NSO are provided in Table 7. Note that these initial ROE correspond to relative motion that begins with a mean along-track separation of -20 [km], a projected circular motion with 300 [m] amplitude in the NR-plane, and a relative drift of approximately 1 [km] per orbit in the along-track direction induced by a nonzero relative semi-major axis. For simplicity but without loss of generality, the VBS on the observing spacecraft is assumed to be mounted with a fixed boresight alignment in the anti-flight direction. Under this assumption, the relative position vectors in the VBS, ($v_{\text{bvs}}$) $\rho$, and RTN frames, ($r_{\text{tn}}$) $\rho$, are related by

\[
[r_{\text{tn}}] = R_{\text{tn}v_{\text{bvs}}}^{V_{\text{bvs}}} [v_{\text{bvs}}] \rho \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} [r_{\text{tn}}] \rho \tag{41}
\]

where $R_{\text{tn}v_{\text{bvs}}}^{V_{\text{bvs}}}$ is the DCM between the RTN and VBS frames.

Beginning from the specified initial conditions, the absolute position and velocity of the observing spacecraft and target NSO are numerically propagated for several orbits using a high-fidelity simulator which includes rigorous force models of high-order gravity, atmospheric drag, solar radiation pressure, third body Sun and Moon effects, and tidal effects.\(^{29}\) The numerically propagated trajectories provide the ground truth against which to compare the performance of angles-only navigation filter. In order to estimate the relative orbit of the NSO, the filter requires knowledge of both the observer’s absolute position and velocity and VBS-frame absolute attitude, as well as sequential sets of bearing angles which subtend the LOS vector.
pointing from the observer to the NSO. The observer absolute orbit knowledge is provided by corrupting the ground truth observer orbit with measurement noise that is representative of coarse Position/Velocity/Time (PVT) solutions obtained using a GPS receiver. Instead, the sensor in the OS testbed loop provides the measured attitude and bearing angles.

The ground truth orbit and attitude of the observer and target are provided to the OS at each time-step in order to render the NSO and a collection of SO on the testbed monitor. Additionally, the target NSO is modeled as a 1U cubesat of side length 10 [cm], with homogeneous planar panels of an assumed reflectance coefficient ρ = 0.2. In accordance with the architecture presented in Appendix A (see Figure 18), all trajectory, attitude, and NSO parameters are used to calculate the scene geometry and radiometry, which are then mapped to the OS monitor through Equations 18, 19, and the empirical mapping depicted in Figure 7. A series of hardware-in-the-loop (HIL) VBS measurements are acquired by the test article described in Section A.2. The bearing angles can be obtained by centroiding the NSO pixel cluster with a digital count weighted average and applying Equations 15-16. Note that the ground truth bearing angles measured by test article are plotted in Figure 16, and their mean and standard deviation over the last three simulated orbits are tabulated in Table 5 (first row). The magnitude of these angular residuals is highly dependent on the angular resolution of the VBS test article, the quality of the OS geometric calibration, and the amount of pixel saturation resulting from modeling the NSO as a multi-variate Gaussian. It is important to note that the worst-case test article azimuth and elevation residuals in Table 5 corresponds to angular errors of less than a quarter of the pixel iFOV (34.2 [arcsec] as noted in Table 3).

The pre- and post-fit measurement residuals (i.e., the difference between the obtained measurement and modeled measurements computed before and after the Kalman filter measurement update) and ROE estimation errors with 1-σ formal standard deviations are shown in Figures 16 and 17, respectively. These results are obtained by providing an adaptive unscented Kalman filter (A-UKF) formulated by Sullivan and D’Amico with the HIL-acquired measurements of the observer attitude and the NSO bearing angles.

A comparison of the angles-only filter pre-fit and post-fit measurement residual steady-state statistics in Table 5 indicate worst-case post-fit residuals for azimuth and elevation at approximately 8% and 22% of the iFOV associated with a test article pixel, respectively. It is instructive to mention that the larger standard deviation in the elevation angle post-fit residuals is expected, since the range ambiguity translates to an elevation error in filter modeling due to the orbit curvature. The filter post-fit trends in Figure 16 indicate large transient residuals in the modeled measurements (particularly for the elevation angle) directly following eclipse periods. Again, this is expected since the modeled azimuth angles following eclipse are conditioned on a state estimate that has been propagated through the entire eclipse without conducting a single measurement update. Still, the subsequent steady-state post-fit elevation residuals account for worst-case angular errors that are less than a quarter of the pixel iFOV. This is a strong indication that the filter is processing measurements effectively and reducing modeling residuals to the noise floor of the onboard sensor.

Similarly, the filter is clearly able to converge to a very good estimate of the relative orbit of the NSO, demonstrating steady-state ROE estimation errors within 1% of their respective ground truth values (see Figure 17). This HIL implementation of angles-only navigation demonstrates the utility of the OS testbed for calibration and verification of VBS and algorithms across a wide swath of the radiometric and geometric operational spectrum.

9. Conclusions

This paper addresses the design, calibration, verification and utilization of a HIL testbed to simulate optical hardware for vision-based navigation in space. The assembled testbed and selected components were converged upon through a design process to meet an explicit set of functional and performance requirements to simulate SO and NSO from a geometric and radiometric stand-point. The OS consists of an OLED monitor

![Figure 15](image.png)

Fig. 15. Relationship between relative position and bearing angles.\(^{11}\)

![Equations 42 and 43](image.png)

\[
\alpha_{\text{truth}} = \arcsin \left( \frac{\rho_y}{|\rho|} \right) \quad (42)
\]

\[
\epsilon_{\text{truth}} = \arctan \left( \frac{\rho_x}{\rho_y} \right) \quad (43)
\]

The relationship between the bearing angles and the relative position is illustrated in Figure 15.

The differences between the ground truth bearing angles and bearings angles measured by test article are plotted in Figure 16, and their mean and standard deviation over the last three simulated orbits are tabulated in Table 5 (first row). The magnitude of these angular residuals is highly dependent on the angular resolution of the VBS test article, the quality of the OS geometric calibration, and the amount of pixel saturation resulting from modeling the NSO as a multi-variate Gaussian. It is important to note that the worst-case test article azimuth and elevation residuals in Table 5 corresponds to angular errors of less than a quarter of the pixel iFOV (34.2 [arcsec] as noted in Table 3).

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### Table 5. Statistics of the VBS and filter residuals for the dynamic relative navigation simulation over the last three simulated orbits. The azimuth and elevation residual means, \(\bar{\alpha}, \bar{\epsilon}\), and 1σ standard deviations are reported in units of [arcsec].

<table>
<thead>
<tr>
<th></th>
<th>(\bar{\alpha} \pm 1\sigma)</th>
<th>(\bar{\epsilon} \pm 1\sigma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VBS</td>
<td>+1.62 ± 0.04</td>
<td>−3.92 ± 0.18</td>
</tr>
<tr>
<td>Pre-Fit</td>
<td>−2.58 ± 36.76</td>
<td>−5.18 ± 58.22</td>
</tr>
<tr>
<td>Post-Fit</td>
<td>−0.89 ± 1.77</td>
<td>+1.03 ± 0.70</td>
</tr>
</tbody>
</table>
stimulating a VBS through a CO. A variety of mechanical deci-
sions were made to facilitate the realization of inter-component
separation, alignment, orientation and interchangeability. Geo-
metric calibration of the testbed consisted of isolating the aber-
rations introduced by the CO and warping the scene rendered
to the monitor to yield a collimated beam of light reaching the
aperture of the VBS test article to be stimulated. Radiometri-
cally, the irradiance and PSF output of the OS was quantified.
The irradiance was modeled as a function of monitor digital
count and number of illuminated pixels. The OS to VBS PSF
transformation was modeled as a first order process governed
by parameters obtained through a Newton-Raphson estimation.
 These calibration steps were necessary to be able to accurately place simulated point sources of light at the intended angular location over a large radiometric dynamic range in the presence of optical aberrations and brightness attenuating mechanisms. Results from the calibration place point sources of light to within tens of arcseconds of angular accuracy while radiometric results demonstrate the OS being able to simulate light sources over eight orders of radiometric magnitude. The quality of the geometric and radiometric calibrations were then verified both geometrically and radiometrically. The geometric verification consisted of a functional comparison against PRISMA flight data and imagery, while the performance verification demonstrated angular residuals between intended and measured point sources of light on the order of tens of arcseconds. The radiometric verification compared experimental results obtained from stimulating a calibration article with the OS against an independent set of measurements acquired from images of the real night-sky.

A series of experiments were conducted to stimulate a VBS test article with synthetic scenes constructed to emulate the space environment in high-fidelity. The first test simulated a static star-field used for inertial navigation. Functional capability of the VBS test article was verified having returned a consistent inertial attitude solution corresponding to the simulated star field. By capturing consecutive images and quantifying the angular deviation between the reported and simulated attitudes, a significant transient characteristic of the sensor strongly correlated to the temperature of the VBS detector was exposed and characterized. After insights gained from this experiment were obtained, the test article was then stimulated by a sequence of a-priori generated star-field images representing scenes that would be perceived by a VBS undergoing a constant angular velocity rotation relative to an inertial reference frame. The synthetically created images were rendered to the OS monitor in an open-loop, temporally regulated fashion via custom written s-functions utilized in the Simulink environment. Results from the HIL acquired imagery provided the ability to characterize the performance of a VBS used in a dynamic inertial spaceborne navigation context in terms of functionality and quality over varying levels of simulated angular velocities. The last conducted experiment consisted of a relative motion simulation used to assess the performance of a relative navigation algorithm that uses VBS measurements collected by a servicer spacecraft performing far-range rendezvous with a noncooperative client in LEO. These HIL observations were used to produce a sequence of inertial attitude measurements as well as relative bearing measurements to the NSO, whose relative position is unknown to the observing vehicle. This vision-based formation flying scenario has a documented observability in discerning the relative separation which can be circumvented through the use of an angles-only filter. The high-dynamic range OS was able to accurately reproduce both SO and NSO from a geometric and radiometric standpoint simultaneously to simulate the VBS test article in a realistic manner. The angles-only relative navigation algorithm was verified by assessing functional performance of the estimation solution and filter measurement modeling accuracy. Future work includes extending the capability of the OS to synthesize and render images in closed-loop and real-time as well as handle close-proximity scenes.

Acknowledgments

The authors would like to express their appreciation to Ronny Votel, Daniel Walker, Adam Koenig, Duncan Eddy, Sumant Sharma, and Josh Egbert who all contributed to the advancement of this testbed. The work was supported in part by the King Abdulaziz City for Science and Technology (KACST), and by the Air Force Research Laboratory’s Control, Navigation, and Guidance for Autonomous Spacecraft (CoNGAS) contract FA9453-16-C-0029. The optical stimulator described in this publication is under provisional patent titled "High Dynamic Range Optical Stimulator for Spaceborne Vision-Based Navigation" with application number 62/413757 and filing date 2016-10-27.

References


Appendix

The architecture of the OS is visually depicted using the flow diagram in Figure 18. This diagram encodes processes/data with small gray/white boxes, respectively. Arrows in the diagram are directional to encode what data are inputs or outputs of a particular process. An arrow entering a process is an input, while an arrow exiting a process block is considered an output. The blue containers represent a subsystem, which is a collection of common processes/data used to achieve a system level objective (i.e. calibration, simulation, stimulation) which is encoded with the larger gray box.

Figure 18 begins with the formation of a VBS calibration article. An independent calibration methodology, such as a star calibration, is necessary to obtain an instrument which report undistorted unit vectors to observed features. The VBS calibration article is then used to quantify the intrinsic parameters of the CO through a residual minimization procedure. The irradiance exiting the CO is quantified using a high resolution optical power meter. These irradiance measurements are then used to construct mappings from OS monitor digital count to desired irradiance reaching the aperture of the VBS test article. The distribution of the light source is matched through a calibration procedure presented in the text. These calibration parameters are used in conjunction with a high-fidelity simulation and radiometric models to synthesize images to be rendered to the OS monitor.
Table 6. Radiometric quantities from night sky collect compared to OS observations.

<table>
<thead>
<tr>
<th>m</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
<th>4.5</th>
<th>2.49</th>
<th>3.25</th>
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</thead>
<tbody>
<tr>
<td>I_o</td>
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<td>1.98e-10</td>
<td>1.25e-10</td>
<td>7.93e-11</td>
<td>5.01e-11</td>
<td>3.16e-10</td>
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<td>ΣDC</td>
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<td>2033248</td>
<td>1773088</td>
<td>1560928</td>
<td>1333104</td>
<td>1707296</td>
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<table>
<thead>
<tr>
<th>m</th>
<th>5.0</th>
<th>5.5</th>
<th>6.0</th>
<th>6.5</th>
<th>7.0</th>
<th>4.91</th>
<th>5.67</th>
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</thead>
<tbody>
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<td>5.07e-12</td>
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<td>1.72e-11</td>
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<td>999872</td>
<td>831904</td>
<td>742960</td>
<td>670112</td>
<td>924352</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 7. Initial servicer orbital conditions and mean ROE test case for dynamic simulation.\textsuperscript{11)}

<table>
<thead>
<tr>
<th>Servicer Orbit</th>
<th>a = 7200 [km]</th>
<th>e = 0.001</th>
<th>i = 30°</th>
<th>Ω = 60°</th>
<th>ω = 120°</th>
<th>M_0 = 0°</th>
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</thead>
<tbody>
<tr>
<td>Initial ROE</td>
<td>a_i [m]</td>
<td>a_i [m]</td>
<td>a_e_x [m]</td>
<td>a_e_y [m]</td>
<td>a_i [m]</td>
<td>a_i [m]</td>
</tr>
<tr>
<td>ROE 2</td>
<td>-100</td>
<td>-20,000</td>
<td>300</td>
<td>0</td>
<td>-300</td>
<td>0</td>
</tr>
</tbody>
</table>

Fig. 18. Architecture used to render a scene to the monitor of the OS.